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Wave forces on arrays of floating bodies

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Summary

Previous work on the scattering of an incident wave field by an array of fixed vertical cylinders is extended to calculate the added-mass and damping coefficients for an array of floating axisymmetric bodies. The method is based upon a large spacing approximation where diverging waves are replaced by plane waves. It is shown that, given the scattering and radiation properties of a single body, the interaction effects within an array can be calculated both simply and accurately.

1. Introduction

The continuing development of offshore structures has led to an increasing interest in the hydrodynamic interactions between neighbouring structures due to wave motions. The scattering of an incident wave field by a group of bodies may lead to wave forces on one of the bodies that differ significantly from the forces it would experience if in isolation. Neglecting viscous effects and using linearised water wave theory, a number of authors have computed these interaction effects. Complex body shapes can be handled using finite element or source distribution techniques; however, computations involving multiple bodies can be prohibitively expensive. Fortunately, many offshore structures are supported by fixed or floating axisymmetric elements and the resulting mathematical simplification is of considerable advantage. A number of authors have considered the problem of wave scattering within a group of fixed or floating vertical cylinders. For example, Ohkusu [1] solved the problem using the method of multiple scattering, where each successive scattering event is considered separately, while Matsui and Tamaki [2] used a source distribution method.

Even with the simplification of considering the bodies to be axisymmetric, computation of the interactions within a group of two or three bodies remains quite complex. A significant step in the simplification of the solution procedure was made by Simon [3]. He suggested that a diverging wave, scattered by, or radiated from a body, may be approximated at large distances by a plane wave. The easiest way to envisage this is to consider circular wave crests. For large radii the crests are locally almost straight when considered on an appropriate length scale, namely the wavelength. In addition to the modelling of diverging waves by plane waves, Simon's method neglects the local fields which decay exponentially away from a body.

Simon [3] used his approximate method to calculate the performance of wave-energy devices. Though he had no other solution method available for comparison, Simon did give an order of magnitude estimate of the errors arising from the plane-wave approximation. McIver and Evans [4] (hereinafter referred to as I) used the plane-wave method to make calculations of horizontal wave forces on arrays of fixed, bottom-mounted, surface-piercing vertical cylinders. For this problem an alternative, exact method of solution (within linear theory) is available. In some of the test comparisons in I, the errors in the wave forces calculated using the plane-wave approximation were more than ten per cent when the cylinders were closely spaced. However, it was shown in I that this error could be reduced to about two per cent, or less, by the incorporation of a simple non-plane correction term which involves very little additional effort. In fact, the use of this correction term for horizontal forces is necessary to obtain the same degree of accuracy given by the plane-wave approximation for vertical forces (see Appendix I).

It is a feature of the scattering problem for the fixed vertical cylinder considered in I that there is no specifically load field decaying exponentially with distance from the body. In general such fields are present, but in the plane-wave method their influence on neighbouring bodies is neglected. This has been shown to be valid in two dimensions (e.g. Srokosz and Evans [5]), even when the body spacing is small. One aim of the present work is to test the plane method when local fields are present in the full linear problem. The situation considered is that of a pair of floating docks for which accurate computations have been made by Matsui and Tamaki [2].

The main purpose of the present work is to provide a more rigorous test of the modified plane-wave method as described in Paper I and to show how it can be applied to more general problems. In Section 2 the mathematical problem is formulated whilst in Section 3 the coupling between a single body and the waves is described. The method of solution for the scattering problem is given in Section 4 and the expressions for the added-mass and damping coefficients for an array of bodies are derived in Section 5. The results for the specific problem of a floating dock are presented in Section 6 and a comparison made with the work of Matsui and Tamaki [2].

2. Formulation

Consider a group of N identical bodies floating in water of depth d. The geometry of an individual body will not be prescribed as yet, though it will be assumed to be vertically axisymmetric. The coordinate system adopted here is that used in I. Thus, Cartesian coordinates are chosen with the x- and y-axes in the horizontal plane of the bottom and the z-axis directed vertically upwards. A sketch of a horizontal section is given in Fig. 1. The j^{th} body has its centre of cross-section at the point (x_j, y_j); relative to this point the field point has coordinates (r_j, θ_j), where θ_j is measured clockwise from the positive y-axis. The centre of the k^{th} body has coordinates (R_{jk}, α_{jk}) relative to the j^{th} body.

The usual assumptions of linearised water-wave theory are made; i.e. the fluid is taken to be inviscid and incompressible and the motion to be irrotational with particle motions of small amplitude. The fluid motion may then be described by a velocity potential $\Phi(x, y, z, t)$ satisfying Laplace's equation within the fluid and the boundary conditions of no flow through solid boundaries, the linearised free-surface condition and a radiation condition of outgoing waves at large distances. The motion is also assumed to be time-harmonic with radian frequency ω; Φ will therefore be written as

$$\Phi(x, y, z, t) = \mathrm{Re}\{\phi(x, y, z)\, \mathrm{e}^{-\mathrm{i}\omega t}\}. \tag{1}$$

All references to the velocity potential of the flow in the following will concern the complex-valued, time-independent, function $\phi(x, y, z)$.

3. Scattering and radiation by a single body

Before considering the interactions within an array of bodies it is necessary to obtain a description of the coupling between the waves and an individual body. The procedure adopted follows closely that used by Simon [3]. The oscillations of the bodies will be restricted to translational motions in the vertical and horizontal directions. Each mode of motion will be denoted by a superfix p; surge motion (parallel to the y-axis) will be denoted by $p = 1$, sway motion (parallel to the x-axis) by $p = 2$ and heave motion by $p = 3$.

Suppose that the j^{th} body oscillates in the p^{th} mode with an amplitude $\xi_j^p \bar{\xi}$, where $\bar{\xi}$ is a typical amplitude of oscillation. Define a complex number $D^{(p)}$ (dependent on frequency)

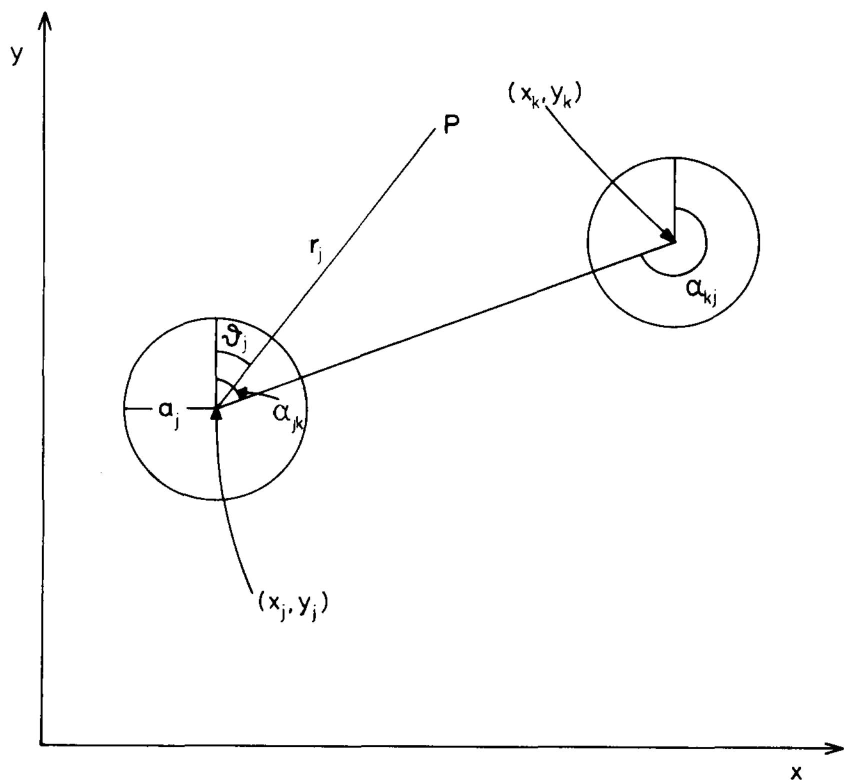

Figure 1. Definition sketch.

for each mode of oscillation by assuming that the radiation potential takes the form

$$\psi_j^p(r_j,\theta_j)=\begin{cases}(Ka)^2D^{(1)}\xi_j^1 f(z)H_1(Kr_j)\cos\theta_j, & p=1 \qquad (2a)\\ (Ka)^2D^{(2)}\xi_j^2 f(z)H_1(Kr_j)\sin\theta_j, & p=2 \qquad (2b)\\ (Ka)^2D^{(3)}\xi_j^3 f(z)H_0(Kr_j), & p=3 \qquad (2c)\end{cases}$$

in the far field. Here, a is a typical radius of the body,

$$f(z)=-\frac{g\bar{\xi}}{\omega}\frac{\cosh Kz}{\cosh Kd}, \tag{3}$$

K is the positive real root of

$$\omega^2=gK\tanh Kd, \tag{4}$$

and H_n is the Hankel function of the first kind and order n. Define a second complex number $E^{(p)}$ (also dependent on frequency) by assuming that the exciting force on the fixed j^{th} body in the p^{th} direction, due to an incident plane wave of amplitude A, may be written as

$$X_j^p=\pi\rho g a^2 A E^{(p)} \tag{5}$$

where ρ and g are respectively the fluid density and the accelertion due to gravity. For $p=1$, 2 X_j^p is to be measured in the direction of wave advance, and for $p=3$ vertically. The constants $D^{(p)}$, $E^{(p)}$ are determined by solving the radiation and plane-wave scattering problems for a single body. Clearly, if the body is axisymmetric then $D^{(1)}$ equals $D^{(2)}$ and $E^{(1)}$ equals $E^{(2)}$.

As noted by Simon [3] the constant $D^{(p)}$ is related to the corresponding $E^{(p)}$. Let the potential ϕ_j represent the scattering by the j^{th} body of a plane wave of amplitude A incident from the direction $\theta_j=\chi+\pi$ so that in the far field

$$\phi_j=f(z)\frac{A}{\bar{\xi}}\sum_{n=0}^{\infty}\epsilon_n \mathrm{i}^n\left(J_n(Kr_j)+A_nH_n(Kr_j)\right)\cos n(\theta_j-\xi). \tag{6}$$

Green's theorem applied to ϕ_j and ψ_j^p over a surface comprising the wetted area of the body, the free surface, the horizontal bottom and an enclosing cylinder in the far field gives

$$D^{(p)}=-\tfrac{1}{2}\pi E^{(p)}\frac{\sinh 2Kd}{2Kd+\sinh 2Kd},\quad p=1,2, \tag{7a}$$

and

$$D^{(3)}=-\tfrac{1}{2}\pi \mathrm{i} E^{(3)}\frac{\sinh 2Kd}{2Kd+\sinh 2Kd}. \tag{7b}$$

Equation (7b) is the generalisation for finite depth of the result given by Simon. In addition, an application of Green's theorem to $\psi_j^p + \psi_j^{p*}$ (* indicates complex conjugate) and ϕ_j gives the results derived by Davis [6],

$$D^{(p)} = D^{(p)*}(1 + 2A_1), \quad p = 1.2, \tag{8a}$$

and

$$D^{(3)} = D^{(3)*}(1 + 2A_0), \quad p = 3. \tag{8b}$$

4. The scattering problem

To determine the added-mass and damping coefficients for the array of bodies, a series of radiation problems must be solved. Each body in turn is forced to oscillate in a particular mode and the resulting hydrodynamic forces on all of the bodies calculated. The radiated wave will be scattered successively by each body in the array. To ease the solution of this complicated scattering problem it will be assumed that the bodies are widely spaced, i.e. for a typical separation distance R, $KR \gg 1$. With this assumption, it was shown in I that a diverging wave travelling away from one body may be approximated in the region of a neighbouring body by a plane wave together with a non-plane correction term.

Suppose a plane wave is incident upon the k^{th} body from the direction $\theta_k = \chi$, the far field potential for the resulting scattered wave may be written

$$\bar{\phi}_k(r_k, \theta_k) = \sum_{n=-\infty}^{\infty} A_n^k \mathrm{i}^n H_n(Kr_k) \exp\{-\mathrm{i}n(\theta_k - \chi)\}. \tag{9}$$

In the vicinity of the j^{th} body this is approximated by a plane wave from the direction of the k^{th} body, with amplitude

$$S_{jk}(\chi) = \bar{\phi}_k(R_{kj}, \alpha_{kj}) = \sum_{m=-\infty}^{\infty} A_m^k \mathrm{i}^m H_m(KR_{jk}) \exp\{\mathrm{i}m(\chi - \alpha_{kj})\}, \tag{10}$$

and a correction term given by

$$D_{jk}(r_j, \theta_j) = \frac{\mathrm{i}}{KR_{jk}} \left\{ \tfrac{1}{2} S_{jk}(\chi) \sum_{n=-\infty}^{\infty} n^2 \mathrm{i}^n J_n(Kr_j) \exp[\mathrm{i}n(\theta_j - \alpha_{kj})] + \mathrm{i}T_{jk}(\chi) \sum_{n=-\infty}^{\infty} n \mathrm{i}^n J_n(Kr_j) \exp[\mathrm{i}n(\theta_j - \alpha_{kj})] \right\} \tag{11}$$

where

$$\mathrm{i}T_{jk}(\chi) = \sum_{m=-\infty}^{\infty} m A_m^k \mathrm{i}^m H_m(KR_{jk}) \exp\{\mathrm{i}m(\chi - \alpha_{kj})\}. \tag{12}$$

The errors in the potential $\bar{\phi}_k$ arising from this approximation are $O((KR)^{-5/2})$, therefore, as pointed out in an addendum to I, it is consistent to simplify S_{jk} and T_{jk} in the first correction term by using the first term in the expansion for large argument of the Hankel function (Abramowitz and Stegun [7], p. 364). Thus

$$S_{jk}(\chi) = \overline{H}_0(KR_{jk}) \sum_{m=-\infty}^{\infty} A_m^k \exp\{im(\chi - \alpha_{kj})\} + O\left((KR)^{-3/2}\right) \tag{13}$$

and

$$iT_{jk}(\chi) = \overline{H}_0(KR_{jk}) \sum_{m=-\infty}^{\infty} mA_m^k \exp\{im(\chi - \alpha_{kj})\} + O\left((KR)^{-3/2}\right) \tag{14}$$

where

$$\overline{H}_0(KR_{jk}) = \left(\frac{2}{\pi KR_{jk}}\right)^{1/2} \exp\{i(KR_{jk} - \tfrac{1}{4}\pi)\}. \tag{15}$$

Where the potential $\overline{\phi}_k$ is to represent a radiated wave it is sufficient to put $\chi = 0$ in Eqns. (9)–(14).

The solution of the scattering problem now proceeds as follows. Let C_{jk} represent the complex amplitude of the plane wave approximation to the *total* wave field incident upon the j^{th} body as a result of scattering and radiation by the k^{th} body. As indicated by equation (10) C_{jk} is found by evaluting the total scattering/radiation potential for the k^{th} body at the coordinate centre of the j^{th} body. Thus, if the m^{th} body oscillates in the q^{th} mode with an amplitude

$$\xi_m^q = \left((Ka)^2 D^{(q)}\right)^{-1}, \tag{16}$$

then

$$C_{jk} = \sum_{l \neq k} C_{kl} S_{jk}(\alpha_{lk}) + \overline{\psi}_m^q(R_{mj}, \alpha_{mj}) \delta_{mk}. \tag{17}$$

The first term on the right-hand side of Eqn. (17) results from the scattering of plane waves by the k^{th} body and $\overline{\psi}_m^k$ is the radiation poential (Eqn. (2)) with the amplitude of oscillation given by Eqn. (16).

In Appendix I it is shown that, with the first correction term applied to the radiated wave only, all wave forces are determined with errors of at most $O((KR)^{-2})$. From Eqn. (5) the wave forces are proportional to the plane wave amplitude, hence errors in C_{jk} of $O((KR)^{-2})$ are admissible. As C_{jk} itself is at most $O((KR)^{-1/2})$, Eqn. (13) may be used to determine $S_{jk}(\alpha_{lk})$ and $\overline{\psi}_m^q$ may be simplified by taking two terms in the large argument expansion of the Hankel function. Thus

$$\overline{\psi}_m^1(R_{mj}, \alpha_{mj}) = -i\overline{H}_0(KR_{jk})\left(1 + \frac{3i}{8KR_{jk}}\right) \cos \alpha_{kj}, \tag{18a}$$

$$\overline{\psi}_m^2(R_{mj}, \alpha_{mj}) = -i\overline{H}_0(KR_{jk})\left(1 + \frac{3i}{8KR_{jk}}\right) \sin \alpha_{kj}, \tag{18b}$$

and

$$\bar{\psi}^3_m(R_{mj}, \alpha_{mj}) = \bar{H}_0(KR_{jk})\left(1 - \frac{\mathrm{i}}{8KR_{jk}}\right). \tag{18c}$$

Note that in I (Eqn. (32)) the radiation potential $\bar{\psi}^q_m$ is replaced by a term representing the scattering of a plane wave incident from outside the wave group. For that term only it would be necessary to include the $O((KR)^{-3/2})$, terms in the expansion of $S_{jk}(\chi)$ given in Eqn. (13).

The $N(N-1)$ equations of the form (17) give a set of simultaneous equations for the complex plane-wave amplitudes which are readily solved by matrix inversion.

5. The added-mass and damping coefficients

Suppose the m^{th} body oscillates in the q^{th} mode with an amplitude given by equation (16), the resulting fluid motion will give an exciting force on the j^{th} body in the p^{th} direction which will be denoted by F^{pq}_{jm}. Decompose this force into components in phase with the acceleration and velocity of the forced oscillation so that

$$F^{pq}_{jm} = \left(\omega^2 A^{pq}_{jm} + \mathrm{i}\omega B^{pq}_{jm}\right)\xi^q_m \bar{\xi}. \tag{19}$$

The real matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are termed respectively the added-mass and damping matrices. Non-dimensional forms will be defined by

$$A^{pq}_{jm} = M\mu^{pq}_{jm}, \tag{20a}$$

$$B^{pq}_{jm} = \omega M\lambda^{pq}_{jm} \tag{20b}$$

where M is the mass of fluid displaced by a single body.

The exciting force on the j^{th} body follows from Eqn. (5) by summing the effects of the incident plane waves together with the first correction terms. A typical plane wave incident from the direction of the k^{th} body will have first correction terms associated with both scattered and radiated waves. It is shown in Appendix I that it is necessary to consider only the first correction for the radiated wave if the wave forces are to have errors $O((KR)^{-2})$. In Appendix II the forces due to the first correction are derived; using Eqns. (A2) and (A3) in (5) gives

$$F^{pq}_{jm} = \pi\rho g a^2 \bar{\xi} E^{(p)} C^p_j \tag{21}$$

where

$$C^1_j = \sum_{k \neq j}\left[C_{jk}\cos\alpha_{kj} + \frac{\mathrm{i}}{KR_{jk}}\left\{\tfrac{1}{2}S^{(q)}_{jk}\cos\alpha_{kj} + T^{(q)}_{jk}\sin\alpha_{kj}\right\}\right], \tag{22a}$$

$$C_j^2 = \sum_{k \neq j} \left[C_{jk} \sin \alpha_{kj} + \frac{\mathrm{i}}{KR_{jk}} \left\{ \tfrac{1}{2} S_{jk}^{(q)} \sin \alpha_{kj} - T_{jk}^{(q)} \cos \alpha_{kj} \right\} \right] \tag{22b}$$

and

$$C_j^3 = \sum_{k \neq j} C_{jk}. \tag{22c}$$

The quantities $S_{jk}^{(q)}$ and $T_{jk}^{(q)}$ are found from Eqns. (13) and (14) using the coefficients appropriate to the far-field radiation potential for oscillations in the q^{th} mode as given by Eqns. (2). Hence,

$$S_{jk}^{(1)} = T_{jk}^{(1)} = -\mathrm{i}\bar{H}_0(KR_{jk}) \cos \alpha_{kj}, \tag{23a}$$

$$S_{jk}^{(2)} = -T_{jk}^{(2)} = -\mathrm{i}\bar{H}_0(KR_{jk}) \sin \alpha_{kj}, \tag{23b}$$

$$S_{jk}^{(3)} = \bar{H}_0(KR_{jk}), \tag{23c}$$

and

$$T_{jk}^{(3)} = 0. \tag{23d}$$

The final expression for the non-dimensional added-mass and damping coefficients is, from Eqns. (19), (20) and (21),

$$\mu_{jm}^{pq} + \mathrm{i}\lambda_{jm}^{pq} = \frac{\pi\rho g a^2}{\omega^2 M} (Ka)^2 D^{(q)} E^{(p)} C_j^p, \tag{24}$$

provided $m \neq j$. When $j = m$, i.e. body j is itself oscillating, the single-body added-mass and damping coefficients must be added to the values given by Eqn. (24). Clearly, there will be no first correction terms in this case.

6. Results

To test the present method a pair of floating docks is considered. Matsui and Tamaki [2] have calculated the added-mass and damping coefficients for such a system using a potentially more accurate source-distribution technique. They considered cylindrical docks of radius $a = 0.1d$ and draught $0.05d$, where d is the water depth. The characteristics of a single body required for the present work were found using the methods of Garrett [7] (the scattering problem) and Yeung [8] (the radiation problem). Checks on the accuracy of these data were made using the relations (7) and (8). In general, the values of $D^{(p)}$ calculated directly from the radiation problem and indirectly from the scattering problem using equations (7) differential by one or two percent at most. The infinite series involving the scattering coefficients A_m^k in equations (13) and (14) were truncated by setting to zero any coefficient less than 10^{-10}.

The comparison with the work of Matsui and Tamaki [2] is made in Figs. 2–4. The curves of Matsui and Tamaki were obtained by tracing photographic enlargements of their original figures. The cross-term added-mass and damping coefficients are presented for sway/sway, heave/heave and sway/heave interactions. That is, for example, the hydrodynamic coefficients given in Fig. 2 describe the inline force on body 1 due to the inline motion of body 2. Results are presented for two separation distances $R_{12} = 3a$ and $R_{12} = 5a$. In general the agreement between the two methods is excellent. Notable discrepancies occur in the added-mass curves for $R_{12} = 3a$ at small wavenumbers. This is

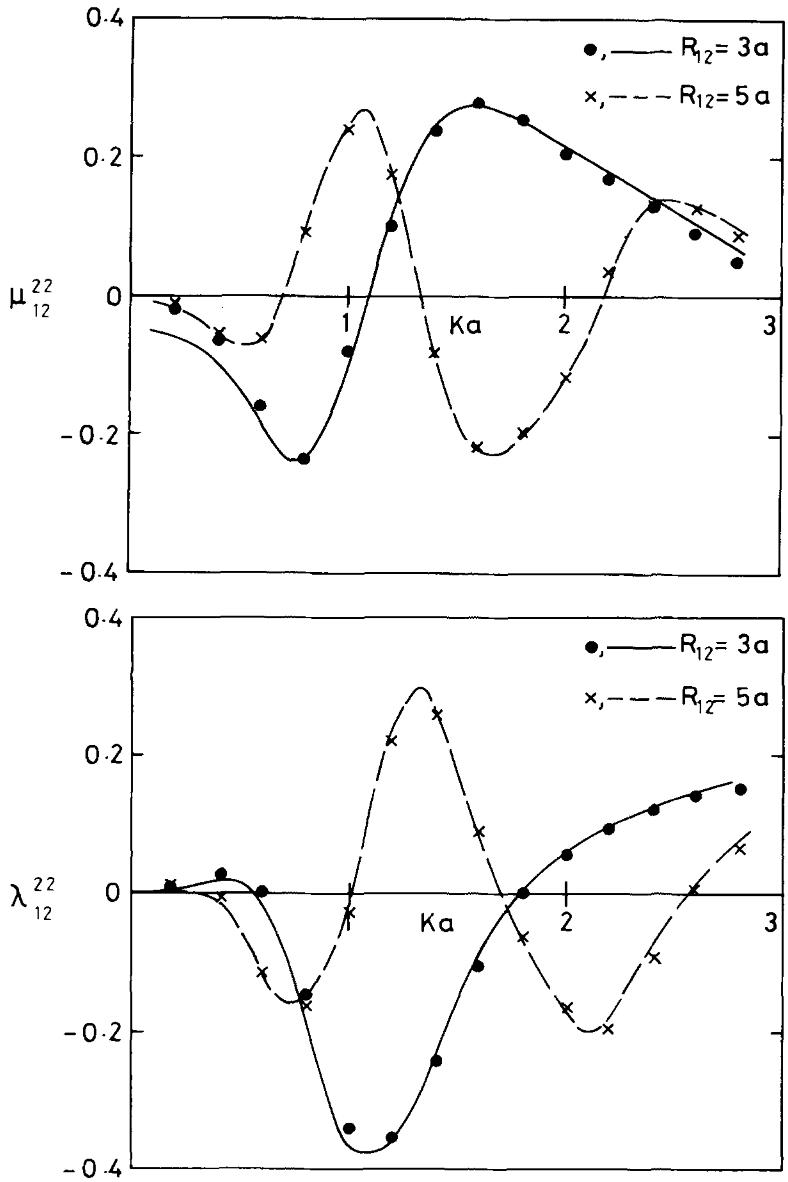

Figure 2. Hydrodynamic coefficients describing inline forces on body 1 due to inline motion of body 2. ×, ●: plane-wave method; —, — — —: Matsui and Tamaki.

not surprising as KR_{12} is then also small and the assumptions of the large spacing approximation are violated. Indeed, it is remarkable that the agreement is generally good for $KR_{12} \sim 1$, confirming the results of I. The discrepancies in the heave/heave added mass results for $R_{12} = 3a$ (Fig. 3a) have no obvious explanation.

The combined results of the present work and I strongly suggest that the plane-wave method gives accurate results for the interactions within groups of axisymmetric bodies for

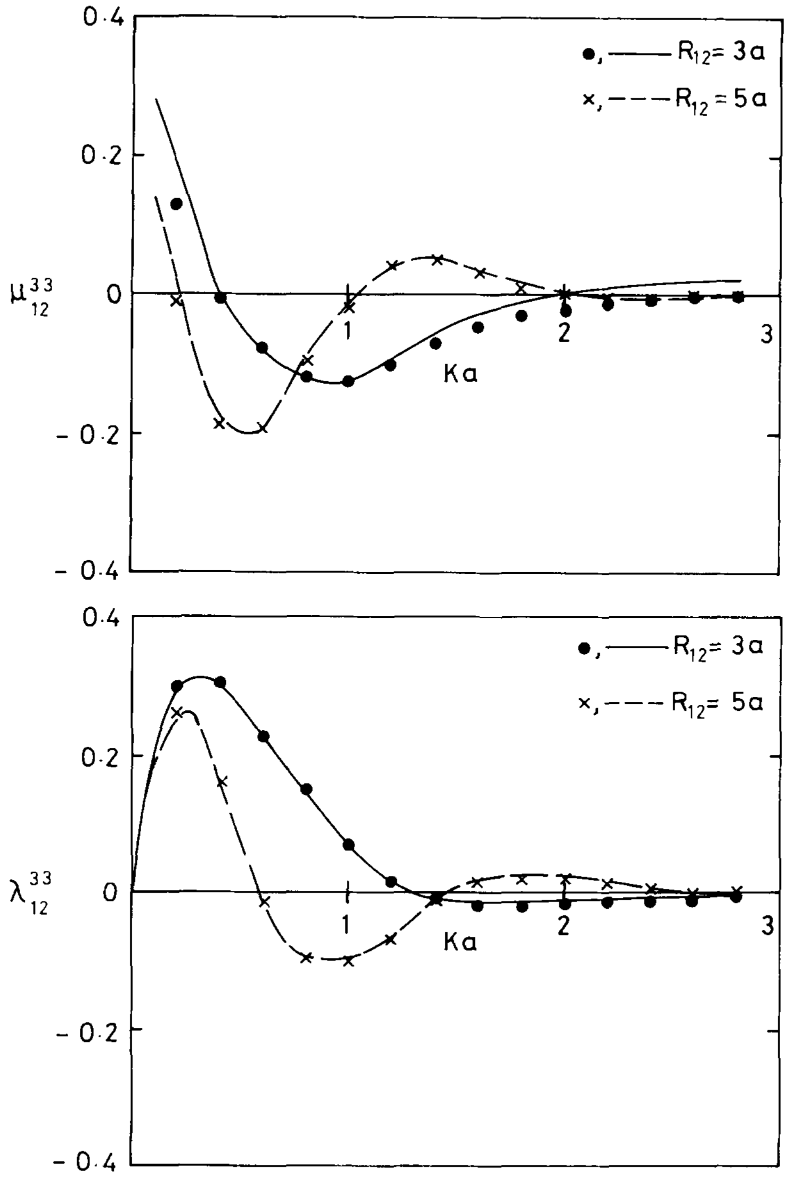

Figure 3. Hydrodynamic coefficients describing the heave forces in body 1 due to the heave motion of body 2. ×, •: plane-wave method; —, — — —: Matsui and Tamaki.

most useful parameter ranges. Provided the scattering and radiation properties of a single body are known, the calculation of the interaction effects is quite straightforward. The procedure involves the summation of series of elementary functions and the inversion of a single matrix equation. The present work has considered a body shape where the single-body characteristics are readily calculated. However, in principle, the necessary data may be obtained for any axisymmetric body using, for example, a source-distribution method.

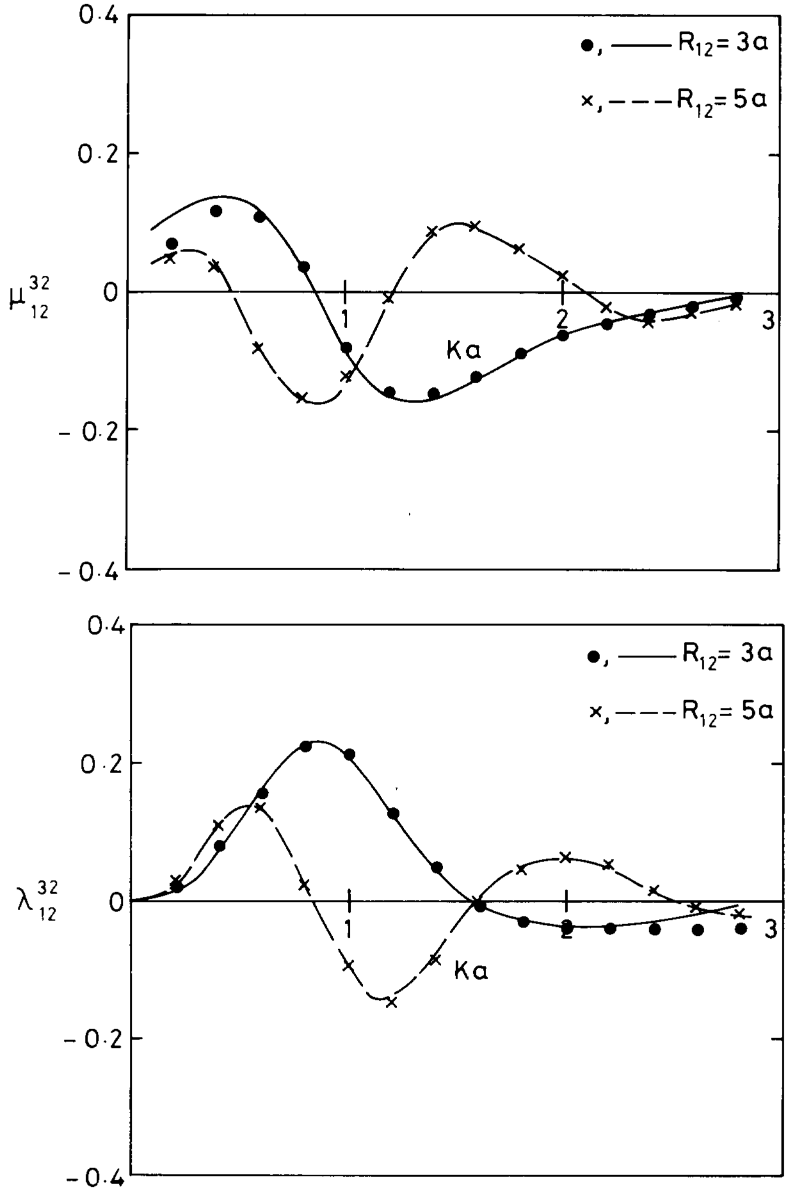

Figure 4. Hydrodynamic coefficients describing the heave forces on body 1 due to the inline motion of body 2. ×, ●: plane-wave method; —, — — —: Matsui and Tamaki.

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Appendix I: Errors arising from the large spacing approximation

For simplicity consider the case of two bodies. Suppose a wave field emanates from body I having a potential given by Eqn. (9). Equations (10–12) are used to approximate the wave field in the region of body 2 with errors in the potential of $O(\zeta^{-5/2})$, where $\zeta = KR_{12}$. The heave force on the body 2 is proportional to the $n = 0$ term while the inline horizontal force is proportional to the difference in the $n = \pm 1$ terms; both are $O(\zeta^{-1/2})$. As noted by Simon [3], the heave force is given exactly by the plane wave approximation only. The horizontal force, however, has errors $O(\zeta^{-5/2})$ provided the first correction term is included. If the plane-wave amplitude, Eqn. (10), is expanded to the second term in the large-argument expansion of the Hankel function (giving the same accuracy as the first correction) the errors in the heave force are also $O(\zeta^{-5/2})$.

The wave field incident upon body 2 is back-scattered to give a force on body 1. In his Appendix, Simon [3] showed that the heave force on body 1 is $O(\zeta^{-1})$, with errors of $O(\zeta^{-2})$ if the plane-wave approximation only is used. A similar calculation shows that the inline force is also $O(\zeta^{-1})$ with errors $O(\zeta^{-2})$. A further scattering of the wave field by body 1 leads to forces on body 2 of $O(\zeta^{-3/2})$ with errors of $O(\zeta^{-5/2})$ if the plane-wave approximation is used. Each successive scattering across the system leads to forces reduced in magnitude by a factor of $O(\zeta^{-1/2})$, hence further scattering lead to forces of the same size as the previously obtained errors and need not be considered.

In summary, the forces on body 2 before scattering are $O(\zeta^{-1/2})$ with errors $O(\zeta^{-5/2})$ when the first correction is used. Accounting for scattering gives additional forces of $O(\zeta^{-3/2})$ with errors of $O(\zeta^{-5/2})$ using the plane-wave approximation only. The forces on body 1 are $O(\zeta^{-1})$ with errors $O(\zeta^{-2})$ from the plane-wave approximation. Hence, for the radiation problem, the wave forces can be found to within errors of $O(\zeta^{-2})$, or less, using the plane-wave approximation provided the first correction is applied to the radiated wave before scattering occurs.

Appendix II: Wave forces due to the first correction

It is assumed that the force on a single body due to an incident plane wave is known, i.e. the constants $E^{(p)}$ of Eqn. (5) are known. Consider the first correction term given by equation (11). This is a non-plane wave field, however, the following argument shows that the vertical and horizontal forces on an axisymmetric body can be deduced immediately once the forces due to a plane wave are known.

Because of the orthogonality properties of the exponential functions in the expansion of D_{jk}, for an axisymmetric body, the vertical force depends only on the $n = 0$ term while the horizontal forces depend on the $n = \pm 1$ terms. The first correction has no $n = 0$ term and so does not contribute to the vertical force. Now, D_{jk} has the same $n = \pm 1$ components as the potential

$$D'_{jk} = \frac{\mathrm{i}}{KR_{jk}} \left\{ \tfrac{1}{2} S_{jk}(\chi) \sum_{n=-\infty}^{\infty} \mathrm{i}^n J_n(Kr_j) \exp\{\mathrm{i}n(\theta_j - \alpha_{kj})\} + T_{jk}(\chi) \sum_{n=-\infty}^{\infty} \mathrm{i}^n J_n(Kr_j) \exp\{\mathrm{i}n(\theta_j - \alpha_{kj} + \tfrac{1}{2}\pi)\} \right\} \tag{A1}$$

which is a combination of plane waves travelling in the directions $\theta_j = \alpha_{kj}$ and $\theta_j = \alpha_{kj} - \pi/2$. Thus, if F is the inline force due to an incident plane wave of unit amplitude, the horizontal forces on the j^{th} body due to D'_{jk}, and hence D_{jk}, are

$$X_j^{(1)} = \frac{\mathrm{i}}{KR_{jk}} \left\{ \tfrac{1}{2} S_{jk}(\chi) \cos \alpha_{kj} + T_{jk}(\chi) \sin \alpha_{kj} \right\} F \tag{A2}$$

and

$$X_j^{(2)} = \frac{\mathrm{i}}{KR_{jk}} \left\{ \tfrac{1}{2} S_{jk}(\chi) \sin \alpha_{kj} - T_{jk}(\chi) \cos \alpha_{kj} \right\} F. \tag{A3}$$

by simple resolution of forces.

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